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Coherent effects of a weak neutral current

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If there is a weak neutral current, then the elastic scattering process $\nu + A \to \nu + A$ should have a sharp coherent forward peak just as $e + A \to e + A$ does. Experiments to observe this peak can give important information on the isospin structure of the neutral current. The experiments are very difficult, although the estimated cross sections (about 10^{-38} cm^2 on carbon) are favorable. The coherent cross sections (in contrast to incoherent) are almost energy-independent. Therefore, energies as low as 100 MeV may be suitable. Quasi-coherent nuclear excitation processes $\nu + A \to \nu + A^*$ provide possible tests of the conservation of the weak neutral current. Because of strong coherent effects at very low energies, the nuclear elastic scattering process may be important in inhibiting cooling by neutrino emission in stellar collapse and neutron stars.

There is recent experimental evidence[1] from CERN and NAL which suggests the presence of a neutral current in neutrino-induced interactions. A primary goal of future neutrino experiments is to confirm the present findings and to investigate the properties of the weak neutral current, for example, the space inversion and internal symmetry structure.

Our purpose here is to suggest a class of experiments which can yield information on the isospin structure of the neutral current not obtainable elsewhere. The idea is very simple: If there is a weak neutral current, elastic neutrino-nucleus scattering should exhibit a sharp coherent forward peak characteristic of the size of the target just as electron-nucleus elastic scattering does. In a sense we are talking about measurements of the nuclear form factors of the weak neutral current analogous to the measurements of the nuclear form factors of the electromagnetic neutral current in elastic electron scattering experiments.[2] In fact, for the same nucleus, these form factors should have the same q^2 dependence. Therefore, the size of the cross section or its extrapolated forward value gives information on the structure of the weak current itself. In the simplest case ($S=0$, $Z=N$ nuclei such as He^4 or C^{12}) the strength of the polar-vector isoscalar component of the weak neutral current is measured directly.

Our suggestion may be an act of hubris, because the inevitable constraints of interaction rate, resolution, and background pose grave experimental difficulties for elastic neutrino-nucleus scattering. We will discuss these problems at the end of this note, but first we wish to present the theoretical ideas relevant to the experiments.

Although the weak neutral current finds a natural place in the beautiful unified gauge theories,[3] it is important to interpret experimental results in a very broad theoretical framework.[4] We assume a general current-current effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \frac{1}{\sqrt{2}} G l^{\mu} \mathcal{J}_{\mu} , \qquad (1)$$

which is consistent with the early findings[1] but far from established. An intermediate neutral vector boson could be included here without affecting the analysis of the low-momentum-transfer processes we are interested in.

The currents will first be written in their fundamental form as they would occur, for example, in particular unified gauge models of the weak, electromagnetic, and strong interactions. We will then write an expression which is essentially model-independent and sufficiently general to parameterize realistic experiments.

To begin with, we write the neutrino current as

$$l'_{\mu} = \bar{\nu}\gamma_{\mu}(1 - \alpha_{\nu}\gamma_5)\nu , \qquad (2)$$

where $V - A$ coupling is not assumed. The hadronic current is assumed to be a sum of components, each corresponding to a symmetry of strong interactions. For example, in a model with the Glashow-Iliopoulos-Maiani (GIM) mechanism,[5] one would have

$$\mathcal{J}'_{\mu} = b(J^{B}_{\mu} + \alpha_B A^{B}_{\mu}) + y(J^{Y}_{\mu} + \alpha_Y A^{Y}_{\mu}) + c(J^{c}_{\mu} + \alpha_c A^{c}_{\mu}) + t\,(J^{I=1,I_3=0}_{\mu} + \alpha_I^{I=1,I_3=0} A^{I=1,I_3=0}_{\mu}) ; \qquad (3)$$

that is one would have a linear combination of baryon number, hyperchange, charm, and third component of isospin. We assume that the polar-vector currents are conserved and normalized (at zero momentum transfer) to the corresponding quantum numbers.

Realistic experiments are done with the lefthanded neutrinos (and right-handed antineutrinos) from meson and muon decay. Because of chirality conservation, there is no loss in generality in writing

$$l^{\mu}=\bar{\nu}\gamma^{\mu}(1-\gamma_5)\nu \tag{4}$$

and multiplying the hadronic current by the overall factor $\frac{1}{2}(1+\alpha_\nu)$. Further, with data from neutrino reactions involving nucleons or nuclei in the initial state, one cannot distinguish[6] among the three isoscalar components in (3), and it is sufficiently general to write

$$\mathcal{J}_\mu = a_0(J_\mu^{(I=0)}+\alpha_0 A_\mu^{(I=0)}) + a_1(J_\mu^{(I=1)}+\alpha_1 A_\mu^{(I=1)}), \tag{5}$$

where $J_\mu^{(I=0)}$ is a conserved vector current, normalized with respect to baryon number or hypercharge (which are identical for the reactions described above.)

The theoretical situation may be restated as follows: In any particular theoretical model with neutral current parameters $\alpha_\nu, b, y, \ldots$, as in (2) and (3), the coefficients a_0, α_0, a_1, α_1 in (5) can be predicted uniquely. Neutrino-scattering data involving nucleonic or nuclear targets can, in principle, tell us these four numbers, but the individual components $\alpha_\nu, b, y, \ldots$, can never be resolved. Thus (5) is a general model-independent expression, whose parameters strongly constrain any model. Current conservation is the key assumption here. A possible test of this assumption involving quasi-coherent nuclear excitation processes is discussed below.

The coefficients a_0, α_0, a_1, α_1, are extremely important numbers, indeed critical for theories of the weak and strong interactions. We mention two models just for illustration. In the Weinberg model, extended to hadrons[7] (either with or without GIM) $a_0=-\sin^2\theta_W$, $a_1=1-2\sin^2\theta_W$, while Sakurai[4] proposes $a_1=0$ with the entire neutral current coupled to baryon number.

In experiments the coefficient values will be difficult to disentangle from the matrix elements of the component currents. Experimental determination of the coefficients a_0, α_0, a_1, α_1 is perhaps best done with elastic transitions of nucleons and nuclei, where at least the vector form factors are known. For spin-zero nuclei, in particular, the axial-vector currents do not contribute, and, as discussed immediately below, the vector form factors are entirely determined by Z,N and the rms nucleus radius r. We have not been able to think of any other experimental configuration where the parameters a_0 and a_1 can be measured so cleanly.

We now analyze the case of neutrino scattering from an $S=0$, $Z=N=\frac{1}{2}A$ nucleus, where only $J_\mu^{(I=0)}$ contributes and we have the matrix element

$$\langle A(p')|\mathcal{J}_\mu|A(p)\rangle = \frac{1}{(2\pi)^3}\frac{1}{(2E2E')^{1/2}}(p+p')_\mu \times a_0 F^{I=0}(q^2) . \tag{6}$$

The form factor $F^{I=0}(q^2)$ reflects the distribution of protons and neutrons in the nucleus and should have essentially the same shape as the nuclear electromagnetic form factors. For small q^2 it is sufficiently accurate to write

$$F^{I=0}(q^2)=A\,e^{-bq^2} . \tag{7}$$

To make rate estimates we will use the electron-scattering results, writing $b=\frac{1}{6}r^2$ relating the b parameters to rms nuclear radii.[8]

The differential cross section for $\nu+A\to\nu+A$ is

$$\frac{d\sigma}{dq^2}=\frac{G^2}{2\pi}a_0^2A^2e^{-2bq^2}\left(1-q^2\frac{2ME+M^2}{4M^2E^2}\right), \tag{8}$$

where E is the neutrino lab energy, q^2 the momentum transfer, and M the target mass. For $q^2\ll M^2$, a condition which is certainly satisfied over the first decade of fall-off from the forward peak in all nuclei, the equality $q^2=q_R^2$ holds, where q_R is the laboratory recoil momentum of the nucleus. G is the conventional Fermi constant: $G=1.01\times10^{-5}(M_{\text{proton}})^{-2}$.

We estimate the expected observable partial cross section as follows. We assume, perhaps optimistically, that the recoil nucleus can be detected for $q_R>q_{\min}=100$ MeV/c, and that the steep decline of the nuclear form factors makes recoil momenta $q_R>q_{\max}=300$ MeV/c unlikely.

For a range of recoil momentum we integrate (8) and find

$$\sigma(q_{\min}<q_R<q_{\max})=\frac{G^2}{2\pi}a_0^2A^2[f(q_{\min}^2)-f(q_{\max}^2)],$$
$$f(x)\equiv(2b)^{-1}e^{-2bx}[1-(8E^2Mb)^{-1}(2E+M)(1+2bx)]. \tag{9}$$

This cross section is accurately energy-independent for $E>1$ GeV, and decreases slowly with energy for $E<1$ GeV.

For helium, we have $r=1.68\times10^{-13}$ cm, $2b=24.2$ (GeV/c)$^{-2}$, and

$$\sigma(\text{He}^4,\ 100<q_R<300\text{ MeV}/c)=a_0^2\times3.6\times10^{-39}\text{ cm}^2,\quad E>1\text{ GeV}$$
$$=a_0^2\times2.5\times10^{-39}\text{ cm}^2,\quad E=200\text{ MeV} .$$

For carbon, $r=2.42\times10^{-13}$ cm, $2b=50.2$ (GeV/c)$^{-2}$

and

$$\sigma(C^{12},\ 100 < q_R < 300\ \text{MeV}/c) = a_0^2 \times 13.6 \times 10^{-39}\ \text{cm}^2,\quad E > 1\ \text{GeV}$$
$$= a_0^2 \times 11.2 \times 10^{-39}\ \text{cm}^2,\quad E = 200\ \text{MeV}\ .$$

For heavier nuclei the approximate estimates should be scaled upward by $A^{4/3}$. In deuterium, the contribution from the polar-vector current would be about a factor of two below helium, but there are axial-vector current effects which are difficult to estimate.[9]

One possibly important effect which we have not considered here is quasi-elastic neutrino scattering with the nucleus emerging in an excited state. This process would add to the rate of observed recoil nuclei, but may complicate the interpretation of results. If the quasi-elastic processes could be observed, there would be very interesting implications. For example, excitation of the low-lying 0^+ states in light nuclei such as O^{16} or Mg^{24} would provide a direct test of the conservation of the polar-vector part of $\mathcal{J}_\mu$. The transition form factors should vanish as q^2 approaches zero if and only if the current is conserved.

Experimentally the most conspicuous and most difficult feature of our process is that the only detectable reaction product is a recoil nucleus of low momentum. Ideally the apparatus should have sufficient resolution to identify and determine the momentum of the recoil nucleus and sufficient mass to achieve a reasonable interaction rate. Neutron background is a serious problem because elastic $n+A$ cross sections are generally large. Kinematics gives the relation

$$\cot\theta_L = \tfrac{1}{2} q_R \frac{E+M}{ME}\left[1 + \frac{q_R^2(1+2E/M)}{4E^2}\right]^{-1/2} \qquad (11)$$

between lab frame angle to the beam, recoil momentum, and neutrino energy. Under the conditions $q_R \ll M$, $q_R \ll E$, the recoil nucleus emerges close to 90° to the beam. This can provide discrimination against background if the recoil angle can be measured.

Careful consideration of all constraints must be given before the feasibility of these experiments can be determined. This note will serve its purpose if our statement of the theoretical issues stimulates experimenters to give the consideration necessary. Our own naive thinking about the experimental possibilities has included deuterium and helium bubble chambers, mineral oil or liquid helium scintillator tanks, and helium and neon streamer chambers.

There is another important point which may have bearing on the experimental possibilities and on our general picture of neutrino interactions. The coherent cross sections[9] are still quite large at $E = 200$ MeV, whereas the conventional charged-lepton production cross sections decrease rapidly with energy. Therefore, it may be advantageous to perform the elastic scattering experiments with muon neutrinos in the 100-MeV region (accessible at a "meson factory") where that part of the neutron background due to neutrino production is small.

There may be interesting astrophysical effects of the elastic neutrino-nucleus scattering process. At low energies (few MeV) where the nucleus is pointlike, the differential cross section (8) becomes

$$\frac{d\sigma}{dz} \approx a_0^2 \frac{G^2}{2\pi} A^2 E^2 (1+z)\ , \qquad (12)$$

where z is the cosine of the neutrino laboratory scattering angle. Backward scattering vanishes (rigorously for spin-zero nuclei and approximately for other nuclei) because of chirality and angular momentum conservation. The elastic cross section, integrated over angle, is

$$\sigma \approx a_0^2 A^2 (E\ \text{in MeV})^2 \times 1.5 \times 10^{-44}\ \text{cm}^2\ . \qquad (13)$$

In stellar collapse[10] this process may become relevant in regimes where column densities exceed $\rho R \approx 10^{17}$ g/cm^2 if medium-weight nuclei such as Fe^{56} are abundant. The scattering cross section for a 10-MeV neutrino on an $A = 50$ nucleus is

$$\sigma \approx a_0^2 \times 3.7 \times 10^{-39}\ \text{cm}^2\ , \qquad (14)$$

and current experiments[1] suggest that $a_0^2 \approx 0.2 \pm 0.1$ (assuming that a_0 and a_1 are not very different). At a volume density of 10^{13} g/cm^3 the mean free path is about 100 m.

Conventionally, neutrino opacity comes from inverse β decay, where low-energy cross sections depend on nuclear-physics details[11] but may be estimated as $10^{-43\pm1}$ cm^2, and from neutrino-electron scattering, where the cross section on an electron at rest[12] is

$$\sigma \approx 1.7 \times 10^{-44}\ \text{cm}^2 \frac{E^2}{m_e(2E+m_e)}$$
$$\approx 1.7 \times 10^{-43}\ \text{cm}^2 \text{ at } E = 10\ \text{MeV}\ . \qquad (15)$$

Averaged over an electron gas of temperature $kT \gg m_e$ and for $E \gg m_e$, one finds[12]

$$\langle\sigma\rangle \approx 2.5 \times 10^{-44}\ \text{cm}^2 \left(\frac{kT}{m_e}\right)\left(\frac{E}{m_e}\right)$$
$$\approx 2 \times 10^{-42}\ \text{cm}^2 \text{ at } E = 10\ \text{MeV},\ kT = 2\ \text{MeV}\ . \qquad (16)$$

There is negligible neutrino-energy loss in nuclear scattering, but the transport cross section is large since the mean scattering angle is 70°. Most of the electron-scattering cross section (16) comes from large-relative-energy configurations, where there is small neutrino energy loss. Of course, inverse β decay is purely absorptive and instantaneously redeposits neutrino energy in the stellar medium.

Therefore we have a transport cross section due to nuclear scattering which is larger than the conventional transport and absorptive cross sections by a factor of 500 or more. At column densities where conventional mechanisms favor neutrino escape, the increased path length in the star due to multiple nuclear scattering makes absorption more probable, and stellar matter may become opaque to neutrinos at lower than conventional density.

Nuclear scattering may also be relevant to blowoff of the supernova mantle and to neutrino processes in the outer layers of a neutron star which consist of neutron-rich nuclei.[13] Since coherent neutrino-nucleus scattering is a straightforward consequence of a weak neutral current (assuming only $a_0 \neq 0$), a thorough study of these astrophysical speculations is worthwhile.

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